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CMM POSITIONING ERRORS CALIBRATION AND THE UNCERTAINTIES MEASUREMENT USING A HOLE BAR AND REVERSAL TECHNIQUE

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Abstract. *The calibration and uncertainties measurement of coordinate measuring machines have been extensively discussed due to the growing use of these instruments in industry. Beside this, the development of calibration methods for coordinate measuring machine (CMM) using mechanical artefacts enhances the importance of having faster and reliable systems at low cost. Discussions about self-calibration, especially the use of reversal techniques for the determination of straightness, flatness, circularity and orthogonality errors can be easily found in today's technical literature. However, not much or almost nothing is known regarding the application of this measurement technique for positioning errors. Self-calibration is a procedure where only local standards are used as reference and there are measurement uncertainties derived from its application. The uncertainty analysis supplies a quantitative measure of the quality of the obtained result. The present work proposes a method of self-calibration for CMM positioning errors using a hole bar and applying the principles of the reversal techniques. The uncertainties measurement originated from the self-calibration process and their propagation through the developed mathematical model are analysed according to the requirements of the ISO Guide for the Expression of Measurement Uncertainty [ISO, 1993].*

Keywords: Self-calibration, Reversal technique, Uncertainty measurement, CMM

1. INTRODUCTION

Coordinate measuring machines (CMMs) are the most universal measuring instruments for the dimensional metrology. However, due to the complexity and universality of these instruments it is not yet possible to assess the errors associated with most of the measuring tasks for which these instruments as used.

Studies of procedures for geometric errors calibration based on self-calibration have been research objectives of metrologists all over the world. Self - calibration is a procedure where only local standards are used as reference. This procedure itself minimises the costs of the standard calibration and reduces the need of using artefacts with long term dimensional stability.

Calibration systems using the reversal technique, for straightness errors, roundness, squareness and flatness are discussed thoroughly in the literature. However almost nothing has been published regarding positioning errors.

For this work, a hole bar was projected (figure 1), manufactured and finally, measured by a Brown&Sharpe “moving bridge” CMM applying the principles of the reversal technique. A mathematical model was developed to calculate the hole bar errors and the CMM positioning errors. The results of the measurements were then compared with those obtained through the direct calibration of the machine positioning errors (scale errors) with a Laser Interferometer System and with the hole bar errors measured by a Universal Measuring Machine (SIP). Analysis of the uncertainties from the self-calibration process and their propagation through the developed mathematical model were made according to the requirements of the Guide to the Expression of Uncertainty in Measurement (1993).

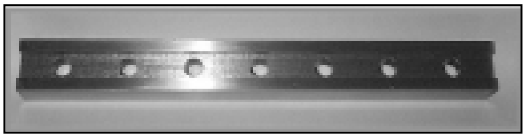

Figure 1: Hole Bar

2. SELF-CALIBRATION MODEL

Classics reversals are all techniques characterised by a mechanical manipulation with respect to a degree of freedom other than the sensitive direction of the indicator. This operation changes the sign of one component of the error.[Evans et al,(1996)].

A variation on these techniques may also be used on measuring machines. The concept is to measure an uncalibrated artefact in multiple positions within the working envelope of the measuring system and from these repeated measurements evaluate the geometric properties of the artefact and the geometric imperfections of the measuring system.

In this work, the model of the machine errors was created using:

- A set of basic functions;
- Assumptions about machine errors.

The procedure is described as follows.

2.1. Analysis of the positioning errors of the hole bar

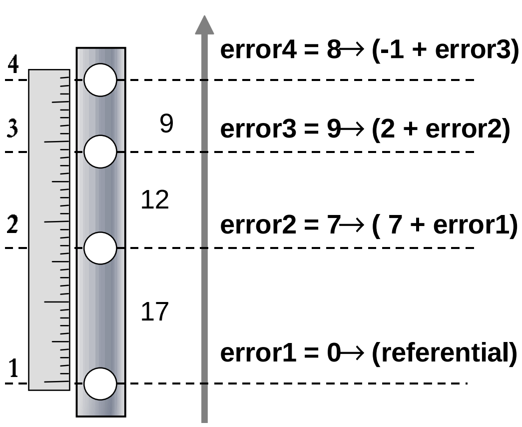

Figura 2: Hole bar errors

The first step to understand the method presented in this work and its associated mathematical model, is the analysis of the positioning errors between the centres of the holes in the bar.

Considering a hypothetical ideal scale, any deviation from those values of the measured distances can be assumed as the hole bar errors. Each error is then a relative error i.e., an error between two consecutive holes, taking always the first hole (n–1) as reference for the next (n), see figure 2.

For example, considering that the nominal distance between holes is always 10, it can be noted that the relative error between holes (1) and (2) is equal to 7, between holes (2) and (3) it is 2 and between holes (3) and (4) it is –1. In figure 2, the grey arrow shows the values of the distances indicated by the scale when the reference is placed at the centre of hole (i-1). Observe that the positioning error at the hole (i) is the same of that between holes (i-1) and (i), plus the positioning error of the hole (i-1) respect to the reference. This is also valid for all the other holes. Generalising:

$$eb_i = eb_{i-1} + (m_i - vm_i) \tag{1}$$

where
eb_i = positioning error at the centre of hole i;
eb_{i-1} = positioning error at the centre of hole i-1;
m_i = measured distance between centres of holes i and i-1 (indicated by the scale)
vm_i = nominal distance between holes as indicated in hole bar drawing;

Changing the reference point from hole (1) to (4) and applying the same analysis, the "Eq.(2)" can be obtained :

$$eb'_{i-1} = eb'_i + (m_i - vm_i) \tag{2}$$

where
eb'_i = positioning error at the centre of hole i (reverse position);
eb'_{i-1} = positioning error at the centre of hole i-1 (reverse position);

2.2. Analysis of the positioning errors of the machine

Considering that an ideal scales do not exists, then the scaling errors should be determined. Assuming the hole bar as a calibrated artefact and the true distances between holes as know then,

$$m_i = m_v + em_i \tag{3}$$

where
m_i = indicated values;
m_v = "calibrated" distances between centres of the holes
em_i = relative errors of the scale

2.3. Deriving the basic equations of the model

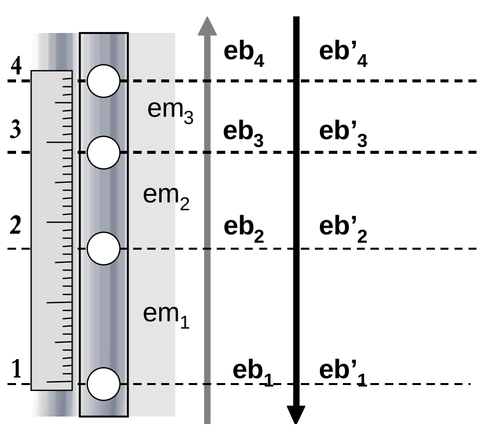

Figura 3: Hole bar errors and machine errors

Substituting the " Eq. (3)" in "Eq. (1) and (2)", one can write:

$$eb_i = eb_{i-1} + (m_i - vm_i) + em_i \tag{4}$$

$$eb'_{i-1} = eb'_i + (m'_i - vm_i) + em_i' \tag{5}$$

which are the basic equations for the proposed model. The model is represented by five groups of equations that are just variations of "Eq. (4) and (5)". Figure 3 shows the errors for a bar of 4 holes.
The change in the referential does not alter the

values $(m_i\text{-}vm_i)$ or em_i, since they are relative errors. Therefore:

$$(m_i - vm_i) = (m_i' - vm_i) \quad \text{and} \quad em_i = em_i'.$$

3. EXPERIMENTAL PROCEDURE AND FINAL MODEL EQUATIONS

The experimental procedure is divided in two parts:

1^{est} part: measurement of the distances between centres of the holes with the reference point placed at the hole (1);

2nd part: the hole bar is rotated 180^o about the normal axis to the measuring plane and the distances between holes are measured again . The reference point is now placed at the centre of the hole (n);

The model equations :

- **1 group of equations:** obtained through the following subtraction:

$$m_i - m_i' = em_i - em_{(i-n)} \text{ for } i = 1,..,n \tag{6}$$

where n is the number of holes, m_i are the measured values in the first stage and mi' are the measured values in the second part. (These equations describes the relationship between the machine errors.)

- **2nd group of equations:** one of the basic equations

$$\begin{aligned} eb_i &= eb_{i-1} + (m_{i-1} - vm_{i-1}) + em_{i-1} \\ eb'_{i-1} &= eb'_i + (m'_i - vm_i) + em_i' \qquad \text{for } i = 1,..,n \end{aligned} \tag{7}$$

- **3rd group of equations:** due to the change in the reference system (second part), the positioning error between the first and the last hole can be expressed as

$$eb_i + eb'_i = eb_n \tag{8}$$

Replacing equations 4 and 5 in equation 8 the following equality can be written

$$(m_i - vm_i + m_{i+1}' - vm_{i+1}) = eb_i + eb'_{i+2} - eb_n - em_i - em_{n-(i+1)} \tag{9}$$

- **4th group of equations:** are model restriction equations:

$$\sum_{i=1}^{n}(m_i - vm_i) = eb_n + \sum_{j=1}^{n-1} em_j \tag{10}$$

$$eb_1' - eb_n = 0 \tag{11}$$

$$eb_1 = 0 \text{ (hole 1 as reference)} \tag{12}$$

- **5th group of equations**: calibrating only half of the hole bar distances, it is possible to guarantee traceability of the measurements when:

$$eb_i + eb_{i+1} = eb_{ci} \qquad \text{for } i = 1 \text{ up to } (n-1)/2 \tag{13}$$

with eb_{ci} being the differences between the calibrated distances and the nominal ones.

4. MEASUREMENT UNCERTAINTY ANALYSIS

According to the *Guide to the Expression of Uncertainty in Measurement* (ISO, 1993), the uncertainty is a parameter associated with the result of a measurement, that characterises the dispersion of the values that could be reasonably attributed to the measurand. This parameter can be for example, a standard deviation or the half-width of an interval having a stated level of confidence.

4.1. How does uncertainty can be classified and calculated ?

In most cases the measurand Y cannot be measured directly, but it is determined from n other quantities Q_1, Q_2,..., Q_n through a functional relationship *f*:

$$Y = f\,(Q_1, Q_2, ..., Q_n) \tag{14}$$

where Q_1,..., Q_n can be measurands too and may also depend on another quantities, including corrections for systematic effects.

The recommendation INC-1 (1980) of the Working Group on Statement of Uncertainties defines the uncertainties associated to the quantities or measurands Q_1, Q_2,..., Q_n in two categories: A and B, taking as basis the evaluation methods and remanding that these categories do not substitute the random or systematic nature of the uncertainty.

- **Uncertainty type A:** when ***statistical methods are applied*** to obtain an estimate of the expected value of a quantity Q_i that varies randomly and for which n independent observations have been obtained. This type of uncertainty is then expressed as the ***standard deviation*** – equation 15 - that characterises the dispersion of the n independent observed values [ISO, 1993].

$$s^2(q_k) = \frac{1}{n-1}\sum_{k=1}^{n}(q_k - \overline{q})^2 \tag{15}$$

In this case, the best available estimate of the expected value of Q_i is the arithmetic mean or average of the n independent observations:

$$\overline{q} = \frac{1}{n}\sum_{k=1}^{n} q_k \tag{16}$$

- **Uncertainty type B:** when the estimate of the quantity Q_i has not been determined through statistical analysis of n observations, the associated uncertainty, expressed as a standard deviation, is estimated ***by scientific judgement*** based on all of the available information about the variability of Q_i. This information includes results of previous measurements, experience or knowledge of the behavior and properties of materials and instruments, maker's data and specifications, data supplied by calibration certificates, related uncertainties in instruction manuals, etc. [ISO, 1993].

It should be said that the estimation of a type B uncertainty can be as reliable as a type A, specially in a measurement situation where the type A uncertainty is obtained from a small number of statistically independent observations.

4.2. Combined Standard Uncertainty

The ISO Guide to the Expression of Measurement Uncertainty (1993) defines a combined standard uncertainty $\boldsymbol{u_c(y)}$ as the positive square root of the combined variance $u_c^2(y)$ given by the equation:

$$u_c^2(y) = \sum_{i=1}^{N}\left[\frac{\delta f}{\delta q_i}\right]^2 u^2(q_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\delta f}{\delta q_i}\frac{\delta f}{\delta q_j}u(q_i, q_j) \tag{17}$$

where
f is the function given by "Eq. (14)" for the estimate y of quantity Y;
u(qi) is a combined or directly evaluated standard uncertainty.

The partial derivatives are considered to be sensibility coefficients, since they determine how much is the contribution of each individual uncertainty source. The combined standard uncertainty characterises therefore the dispersion of the values that reasonably could be attributed to the measurand Y.

The equation 17 is based on a first order Taylor series approach of $Y = f(Q_1, Q_2,..., Q_n)$ and is called as *law of propagation of uncertainty*.

4.3. Uncertainty Analysis of the Self-calibration Model

When the proposed calibration method is applied, the uncertainties involved are:

- those originated from the repetitive distance measurements of the hole bar in the CMM **(type A);**
- the uncertainties associated to the hole bar calibration process. In this case, we know exactly how the uncertainties associated to the calibrated distances were obtained and therefore, it can be said that they are also of **type A**;
- the uncertainties originated from the influence of temperature variations **(type B),** which can be let apart because the machine is in a controlled environment.

The uncertainties associated to the individual coordinate points involved in the geometry (circle) determination and consequently, in the distance calculation is already taken into account the uncertainty associated to the hole centre points. The distances measured are not correlated and therefore, the last term of "Eq. (17)" is not considered.

5. RESULTS

The proposed self-calibration method and the derived errors equations were applied to a Brown&Sharpe “moving bridge” CMM. The distances between the holes centres of the hole bar were first calibrated using a SIP.

The least-squares fitting was used to solve the system.

The figure 4 shows the great agreement found between the positioning error curve of the CMM Y axis measured with a Laser Interferometer System and the positioning error curve for the same axis found using the self-calibration method and the developed model.

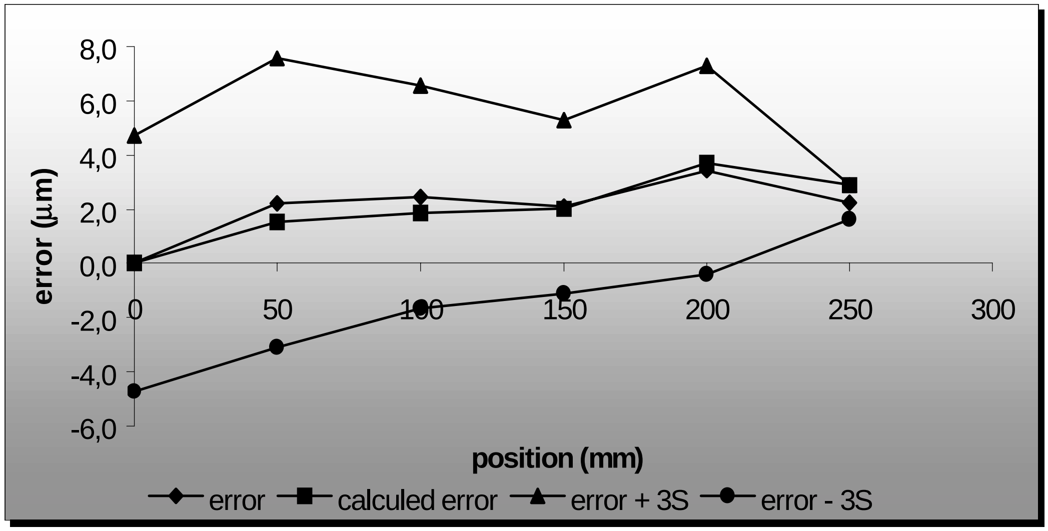

Figure 4: Machine positioning error

The errors of the hole bar found by the calibration (difference between the distances measured with the SIP UMM and the nominal distances) were compared to those obtained for the hole bar with of the equations proposed model. Figure 5 shows the results of this comparison. The major difference found was 3µm.

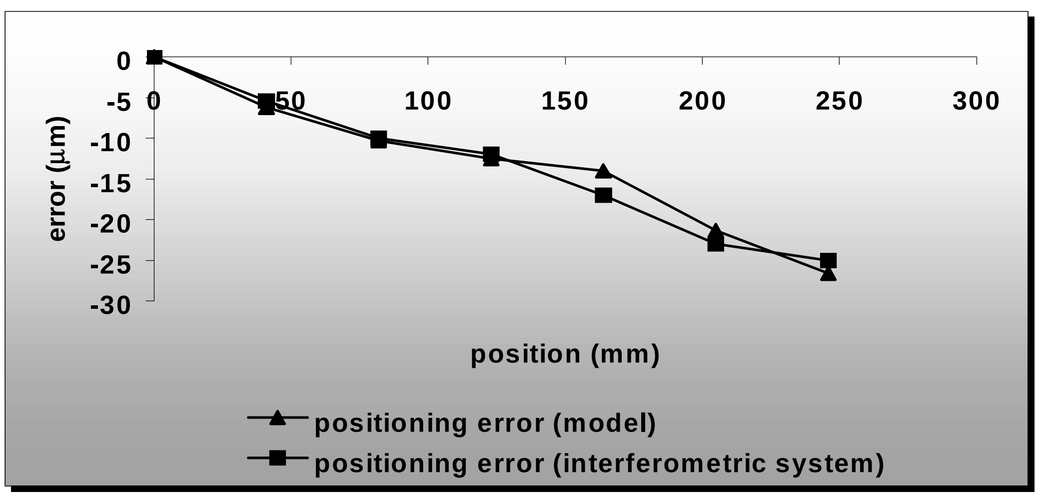

Figure 5: Hole bar positioning errors

Based on analysis proposed (see 4.3.), a study of the error propagation through the developed mathematical model was accomplished using software for symbolic manipulation Table 1 shows the combined standard uncertainties found by the application of the proposed **calibration method**.

Table 1: Combined standard uncertainties

Error	**Error value (μm)**	**u_C(error) (μm)**
eb1	0	0.05
eb2	-6	0.46
eb3	-10	0.05
eb4	-13	0.05
eb5	-14	3.3
eb6	-21	3.9
eb7	-27	4.9
em1	1.5	1.5
em2	1	1.6
em3	-0.5	1.5
em4	0.5	2.6
em5	1.7	3.9
em6	-1	2.2

Since em_j represents the relative error between two consecutive centre points, the calculation of the absolute positioning errors of the machine also propagates uncertainties. Table 2 presents the machine positioning errors found at a different axis positions and the uncertainties associated to them.

Table 2: Machine positioning errors and combined standard uncertainties

Position (mm)	**Error (μm)**	**U_C(error) (μm)**
0	0	0
41	1.5	1.5
82	2.5	2.19
123	2	2.65
164	2.5	3.72
205	4.2	5.39
246	3.2	5.82

It can be observed, that the uncertainty values associated to the positioning errors of the hole bar are larger for larger distances (table 1). This is due to the way it was modelled fact, the error of the n hole depends on the error of the n-1 hole ["Eq.(4) and (5)]". The same occurs with the uncertainties associated to the machine positioning errors.

6. CONCLUSIONS

The uncertainty analysis, despite of requiring a great mathematical effort, it is a requirement of the new concept of traceability necessary for the quantitative analysis of the results. The uncertainty values obtained can be considered as satisfactory.

With the proposed self-calibration method the measurement times are short and the whole process requires least computational efforts. The CMM traceability is guaranteed through the hole bar calibration. Other studies about the traceability associated to this method are still being made with the objective of minimising the number of bar distances that should be calibrated.

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